C 61216

(Pages : 4)

Name..............................

Reg. No..............................

FOURTH SEMESTER B.A./B.Sc. DEGREE EXAMINATION, APRIL 2019

(CUCBCSS—UG)

Common Course for LRP

MA T4 B04—THEORY OF EQUATION, MATRICES AND VECTOR CALCULUS

Time : Three Hours

Maximum : 80 Marks

Part A (Objective Type)

Answer all **twelve** *questions.*
Each question carries 1 mark.

1. If α, β, γ are the roots of $x^3 - px^2 + qx - r = 0$ find the value of $\sum \alpha^2$.

2. Define a reciprocal equation.

3. State Descarte's rule of signs.

4. If α, β, γ are the roots of $f(x) = 0$, write the equation whose roots are $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$.

5. Rank of the matrix $A = \begin{bmatrix} 0 & 2 & 3 \\ 0 & 4 & 6 \\ 0 & 6 & 9 \end{bmatrix}$ is .... .

6. If A is a non-zero column matrix and B is a non-zero row matrix then rank (AB) is .... .

7. The system AX = 0 in n unknowns has a non-trivial solution if ———.

8. For what value of a the system of equations $ax + y = 1, x + 2y = 3, 2x + 3y = 5$ are consistent.

9. If A is an n-rowed non-singular matrix, X and B are $n \times 1$ matrices, then the system of equations AX = B has ——— solution.

10. Find the parametric equation of a line through the point (3, – 4, –1) and parallel to the vector $\mathbf{i} + \mathbf{j} + \mathbf{k}$.

Turn over

11. Find the unit vector tangent to the curve $r(t) = (2\cos t)\,\mathbf{i} + (2\sin t)\,\mathbf{j} + \sqrt{5}t\,\mathbf{k}, 0 \le t \le \pi$.

12. Write the equations relating rectangular and cylindrical co-ordinates.

(12 × 1 = 12 mark

Part B (Short Answer Type)

Answer any **nine** *questions.*
Each question carries 2 *marks.*

13. Solve $x^3 - 12x^2 + 39x^2 - 28 = 0$ whose roots are in arithmetic progression.

14. Transform $x^3 - 6x^2 + 5x + 12 = 0$ into an equation lacking the second term.

15. If $\alpha, \beta, \gamma, \delta$ are the roots of $x^4 + px^3 + qx^2 + rx + s = 0$ evaluate $\sum \alpha^2\beta\gamma$.

16. If $A = \begin{bmatrix} 1 & 5 & 2 \\ 3 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 1 & 4 \\ 2 & 1 \end{bmatrix}$, then rank of AB is :

17. Under what condition the rank of the matrix $A = \begin{bmatrix} 2 & 4 & 2 \\ 3 & 1 & 2 \\ 1 & 0 & x \end{bmatrix}$ is 3.

18. Show that corresponding to a characteristic vector X of a square matrix A, there exist one and only one characteristic root.

19. If A is non-singular, prove that the eigen values of A^{-1} are the reciprocals of the eigen values of A.

20. Show that the characteristic roots of a Hermitian matrices are all real.

21. Find the velocity and acceleration vectors of $r(t) = (t + 1)\,\mathbf{i} + (t^2 - 1)\,\mathbf{j}$ at $t = 1$.

22. Find a spherical co-ordinate equation for the sphere $x^2 + y^2 + (z - 1)^2 = 1$.

23. Evaluate $\int_0^\pi \left((\cos t)\,\mathbf{i} + \mathbf{j} - (2t)\,\mathbf{k}\right) dt$.

24. Find the curvature of $r(t) = t\,\mathbf{i} + (\ln \cos t)\,\mathbf{j}, -\pi/2 < t < \pi/2$.

(9 × 2 = 18 marks)

Part C (Short Essay Type)

Answer any **six** *questions.*
Each question carries 5 marks.

25. If α, β, γ are roots of $x^3 - x - 1 = 0$, find the equation whose roots are $\frac{1+\alpha}{1-\alpha}, \frac{1+\beta}{1-\beta}, \frac{1+\gamma}{1-\gamma}$. Hence write down the values of $\sum\left(\frac{1+\alpha}{1-\alpha}\right)$.

26. If α, β, γ are roots of $x^2 + qx + r = 0$, find the equation whose roots are $(\beta-\gamma)^2, (\gamma-\alpha)^2, (\alpha-\beta)^2$.

27. Solve the equation $x^4 + 6x^3 - 5x^2 + 6x + 1 = 0$.

28. Reduce the matrix A to its normal form and hence find the rank of A where :

$$A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}.$$

29. Show that rank (AA′) = rank (A).

30. Find the latent roots and latent vectors of the matrix $A = \begin{bmatrix} a & h & g \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$.

31. Find the point where the line $x = \frac{8}{3} + 2t, y = -2t, z = 1 + t$ intersects the plane $3x + 2y + 6z = 6$.

32. Find the distance from the point S (1, 1, 5) to the line L : $x = 1 + t, y = 3 - t, z = 2t$.

33. Using matrix method solve the equations :

$$x + y + z = 6$$

$$x - y - z = 2$$

$$2x + y - z = 1.$$

(6 × 5 = 30 marks)

Part D (Essay Type)

Answer any **two** *questions.*
Each question carries 10 ***marks.***

34. Solve the equation $x^3 - 3x^2 + 12x + 16 = 0$ by Cardan's method.

35. Show that the matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ satisfies Cayley-Hamilton theorem. Hence obtain A^{-1}.

36. Find the binormal vector and torsion for the space curve $r(t) = (3 \sin t)\,\mathbf{i} + (3 \cos t)\,\mathbf{j} + 4t\mathbf{k}$.

(2 × 10 = 20 marks)

D 41959 (Pages : 4) Name..............................

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FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2018

(CUCBCSS—UG)

Mathematics

MAT 4B 04—THEORY OF EQUATIONS MATRICES AND VECTOR CALCULUS

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type)

Answer **all the twelve** *questions.*
Each question carries 1 *mark.*

1. If α, β, γ are the roots of $2x^3 + 3x^2 - x - 1 = 0$. Find the equation whose roots are $\frac{1}{2\alpha}, \frac{1}{2\beta}, \frac{1}{2\gamma}$.

2. State the Fundamental theorem of algebra.

3. If α, β, γ are the roots of $ax^3 + 3bx^2 + 3cx + d = 0$. Find the value of :

   $$(\beta-\gamma)(\gamma-\alpha) + (\gamma-\alpha)(\alpha-\beta) + (\alpha-\beta)(\beta-\gamma).$$

4. What do you mean by reciprocal equation of second type ? Give example.

5. What is the rank of the identity matrix of order 20 ?

6. If $A = [a_{ij}]$ is an $m \times n$ matrix and $a_{ij} = 7$ for all i, j then rank of A is ————.

7. A system of m homogeneous linear equations in n unknowns has only trivial solution if ————.

8. For what values of a the system of equations $ax + y = 1, x + 2y = 3, 2x + 3y = 5$ are consistent.

9. If the number of variables in a non-homogeneous system AX = B is n then the system possesses a unique solution if ————.

10. Find the parametric equation of a line through P (3, – 4, – 1) and parallel to the vector $i + j + k$.

11. Find the unit vector tangent to the curve $r(t) = (\cos^3 t)\, j + (\sin^3 j)\, k, 0 \le t \le \frac{\pi}{2}$.

12. Write equations relating rectangular and cylindrical co-ordinates.

(12 × 1 = 12 marks)

Turn over

Part B (Short Answer Type)

*Answer any **nine** questions.*
Each question carries 2 marks.

13. Solve $4x^3 - 24x^2 + 23x + 18 = 0$. Given that the roots are in arithmatic progression.

14. Transform $x^3 - 6x^2 + 5x + 12 = 0$ into an equation lacking second term.

15. If α, β, γ are the roots of $x^3 + qx + r = 0$. Find the equation whose roots are $(\beta - \alpha)^2, (\gamma - \alpha)^2, (\alpha - \beta)^2$.

16. If $A = \begin{pmatrix} -2 & -1 \\ 5 & 4 \end{pmatrix}$. Find A^{-1}.

17. Prove that the characteristic roots of Hermitian matrix are real.

18. If α is an eigen value of a non-singular matrix A, prove that $\frac{|A|}{\alpha}$ is an eigen value of adj A.

19. Show that the product of characteristic roots of a square matrix of order n is equal to the determinant of the matrix.

20. Find the value of a for which $r(A) = 3$ where $A = \begin{pmatrix} 2 & 4 & 4 \\ 3 & 1 & 2 \\ 1 & 0 & a \end{pmatrix}$.

21. Find the velocity and acceleration vectors of $r(t) = (3\cos t)\,i + (3\sin t)\,j + t^2 k$.

22. Find a Cartesian equation for the surface $z = r^2$. And identify the surface.

23. Evaluate $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left[(\sin t)\,i + (1 + \cos t)\,j + \left(\sec^2 t\right) k\right] dt$.

24. Find the normal vector for $r(t) = (a\cos t)\,i + (a\sin t)\,j + bk$.

(9 × 2 = 18 marks)

Part C (Short Essays)

Answer any **six** *questions.*
Each question carries 5 *marks.*

25. If α, β, γ are the roots of $x^3 - x - 1 = 0$. Find the equation whose roots are $\frac{1+\alpha}{1-\alpha}, \frac{1+\beta}{1-\beta}, \frac{1+\gamma}{1-\gamma}$.

26. Solve the equation $x^2 - 12x - 65 = 0$ by Cardan's method.

27. Solve $x^3 + 6x^2 + 3x + 18 = 0$.

28. Prove that the rank of a non-singular matrix is equal to the rank of its reciprocal matrix.

29. Find the rank of $\begin{pmatrix} 4 & -2 & 3 \\ 2 & 0 & 1 \\ 3 & -1 & 2 \\ 1 & -1 & 1 \end{pmatrix}$.

30. Using matrix method solve :

$$x + 2y + z = 2$$
$$3x + y - 2z = 1$$
$$4x - 3y - z = 3$$
$$2x + 4y + 2z = 4.$$

31. Find the point in which the line $x = 1 - t, y = 3t, z = 1 + t$ intersects the plane $2x - y + 3z = 6$.

32. Find the distance from the point S (0, 0, 1, 2) to the line $x = 4t, y = -2t, z = 2t$.

33. Find the eigen values and eigen vectors of $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$.

(6 × 5 = 30 marks)

Part D

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. Solve the equation $x^5 - 5x^4 + 9x^3 - 9x^2 + 5x - 1 = 0$.

35. Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & -2 & 1 \\ 2 & 1 & 1 \\ 0 & 5 & -1 \end{pmatrix}$ and hence evaluate A^{-1}.

36. Find the binormal vector and torsion for the space curve $r(t) = \left(e^t \cos t\right) i + \left(e^t \sin t\right) j + 2k$.

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